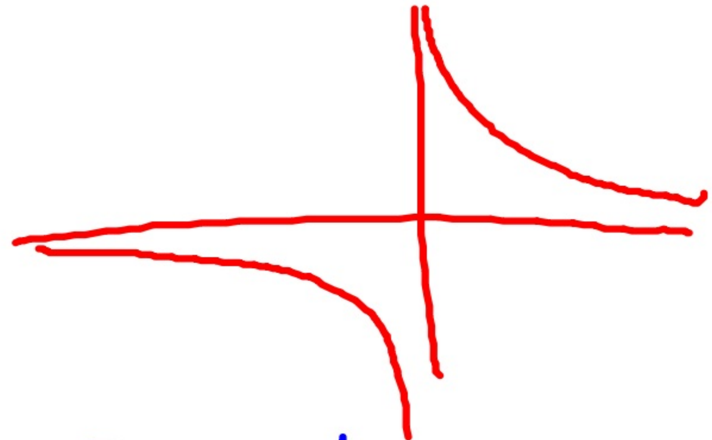
Page 75, Numbers 1 - 6 all

Divide using polynomial long division or synthetic division

$$x^3 - 1 \quad / \; x - 1$$

$$
\begin{array}{r l}
 & x^2 + x + 1 \\
x - 1 \,\big) & x^3 + 0x^2 + 0x - 1 \\
 & \underline{-x^3 + x^2} \\
 & \qquad x^2 + 0x - 1 \\
 & \qquad \underline{-x^2 + x} \\
 & \qquad\qquad x - 1 \\
 & \qquad\qquad \underline{-x + 1}
\end{array}
$$

Section 2.2
Limits Involving Infinity



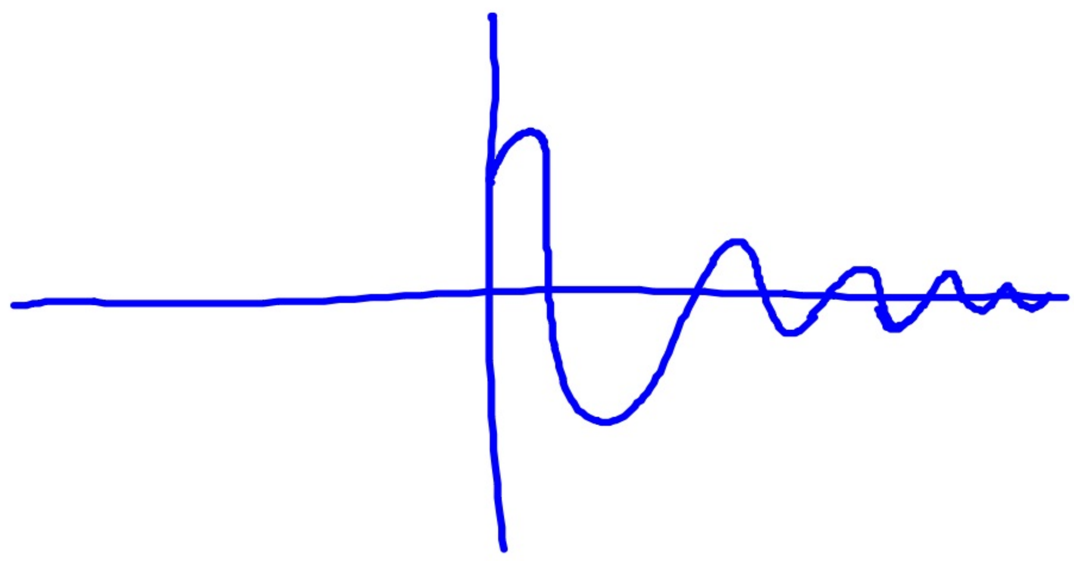
$$\lim_{x \to \infty} (1/x) = 0 = \frac{1}{\infty}$$

$$\lim_{x \to -\infty} (1/x) = 0$$

Horizontal Asymptote

The line y = b is a horizontal asymptote of the graph of a function, y = f(x) if either

$$\lim_{x \to \infty} f(x) = b \quad \text{or} \quad \lim_{x \to -\infty} f(x) = b$$



Example

Find the horizontal asymptote of

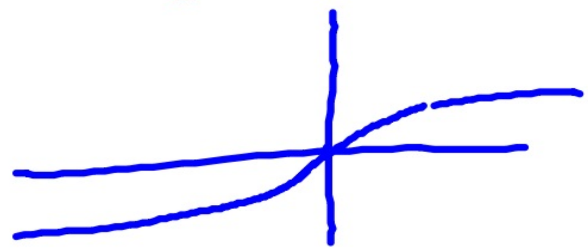
f(x) = 2 + (1/x)

$$\lim_{x \to \infty} 2 + \frac{1}{x} = 2 + 0 = 2$$

Example

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2}} = \frac{x}{x} = 1$$

Use graphs and tables to find $\lim_{x\to\infty} f(x)$, $\lim_{x\to -\infty} f(x)$, and identify all horizontal asymptotes of



$$f(x) = x/\sqrt{x^2+1}$$

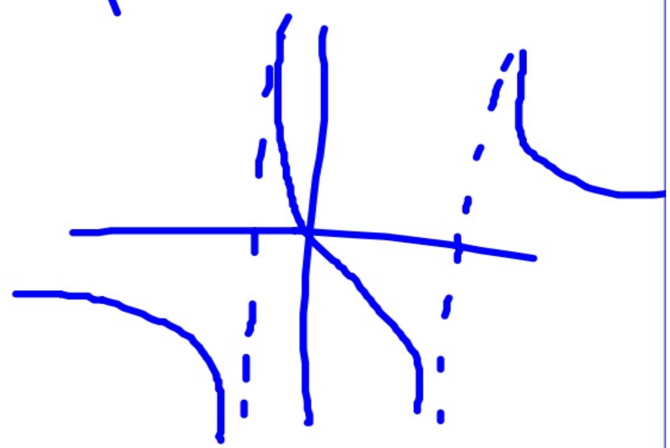
$$\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}} = \frac{-x}{+x} = -1$$

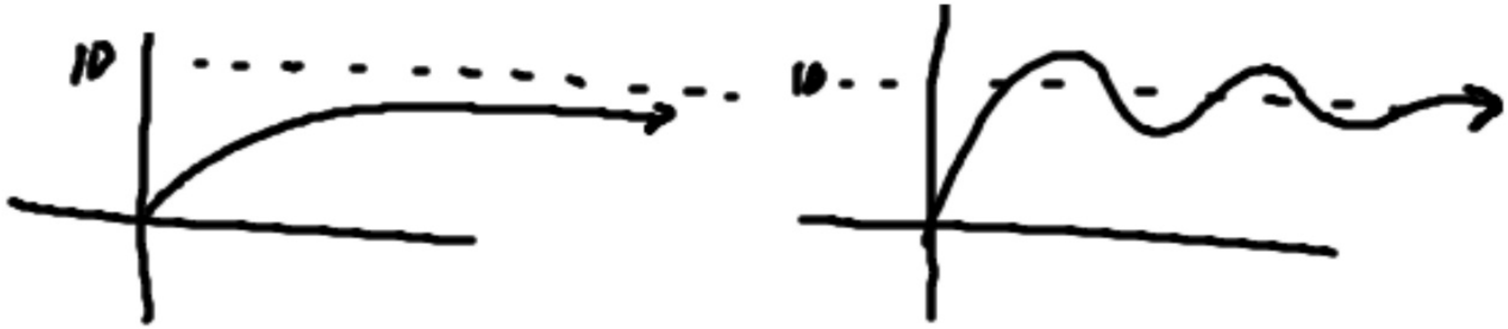
Does $\dfrac{2x + 1}{|x| - 3}$ have any horizontal asymptotes?

$$\lim_{x\to\infty} \frac{2x+1}{|x|-3} = \lim_{x\to\infty} \frac{2x}{|x|}$$

$$= \lim_{x\to\infty} \frac{2\cancel{x}}{\cancel{x}} = 2$$

$$\lim_{x\to-\infty} \frac{2x}{|x|} = -2$$





$$\lim_{x \to \infty} f(x) = 10$$

Find the vertical asymptotes of

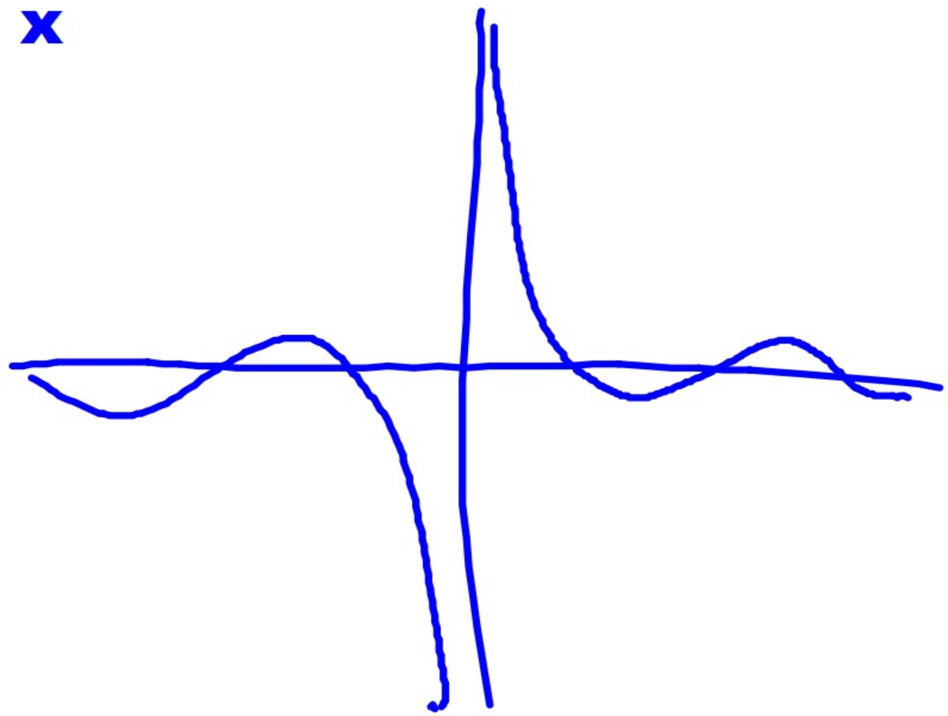
$$f(x) = \frac{x^2 - 1}{x+1} = \frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} = x - 1$$

$$\lim_{x \to -1} \frac{x^2 - 1}{x+1} = \lim_{x \to -1} (x-1)$$

$$= -2$$

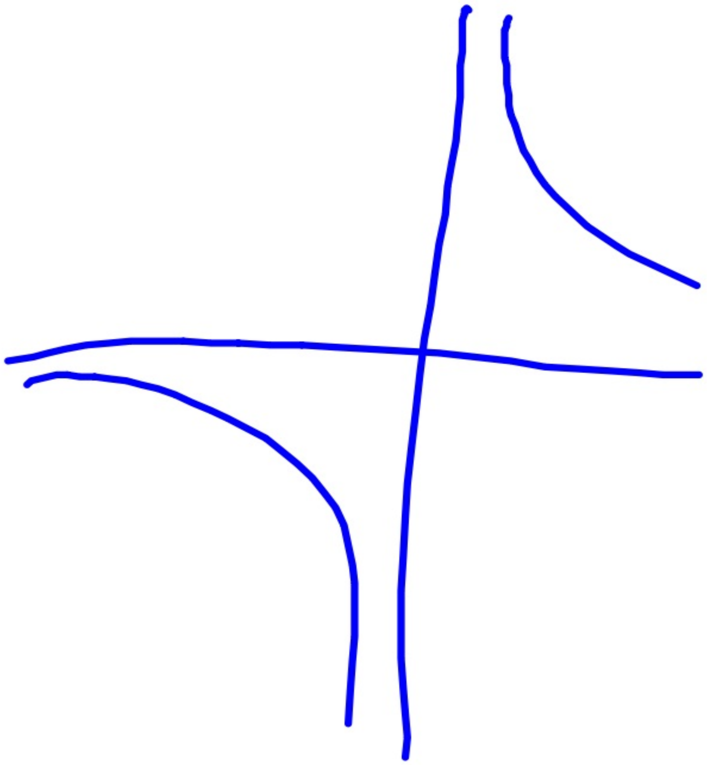
Find the vertical asymptotes

$$f(x) = \frac{\cos x}{x}$$



$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$



Vertical Asymptote

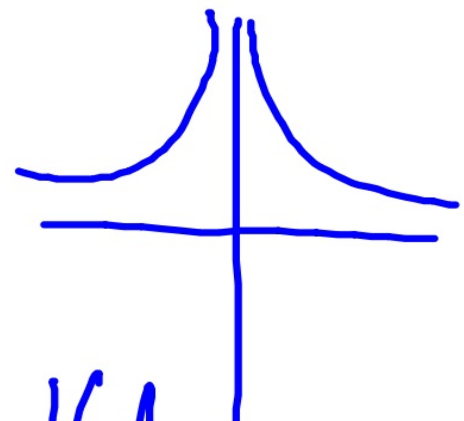
The line x = a is a vertical asymptote of the graph of a function y = f(x) if either

$$\lim_{x \to a^+} f(x) = +/- \infty \quad \textbf{or} \quad \lim_{x \to a^-} f(x) = +/- \infty$$

Find the vertical asymptotes of $f(x) = \frac{1}{x^2}$

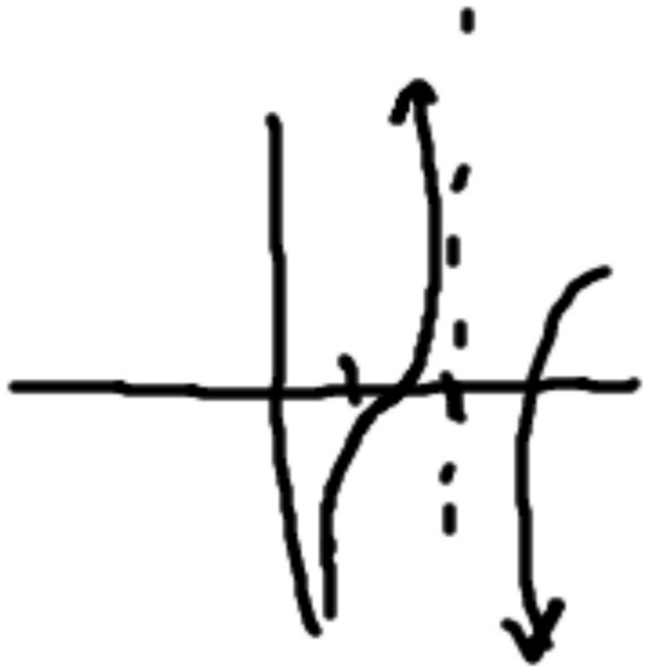
Describe the behavior to the left and right of each vertical asymptote

$$\lim_{x \to 0^+} \frac{1}{x^2} = \infty$$



$$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$$

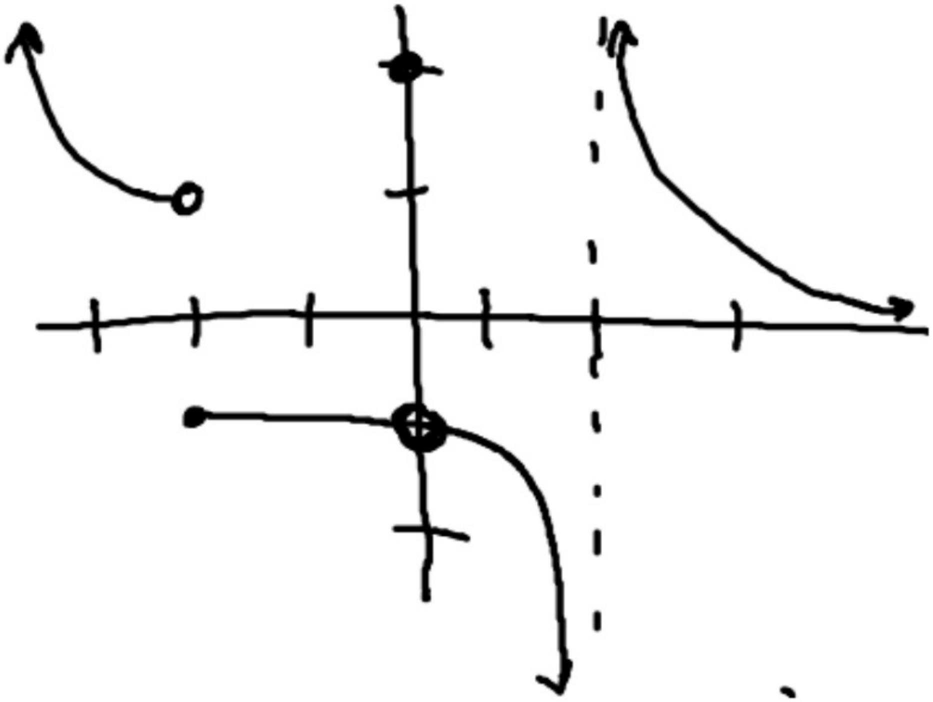
$\therefore$ VA is $x = 0$



$\lim_{x \to 2^+} f(x) = -\infty$

$\lim_{x \to 2^-} f(x) = \infty$

$\lim_{x \to 2} f(x) =$ DNE



1.) $\lim_{x \to -2^-} f(x) =$ 1

2.) $\lim_{x \to -2^+} f(x) =$ −1

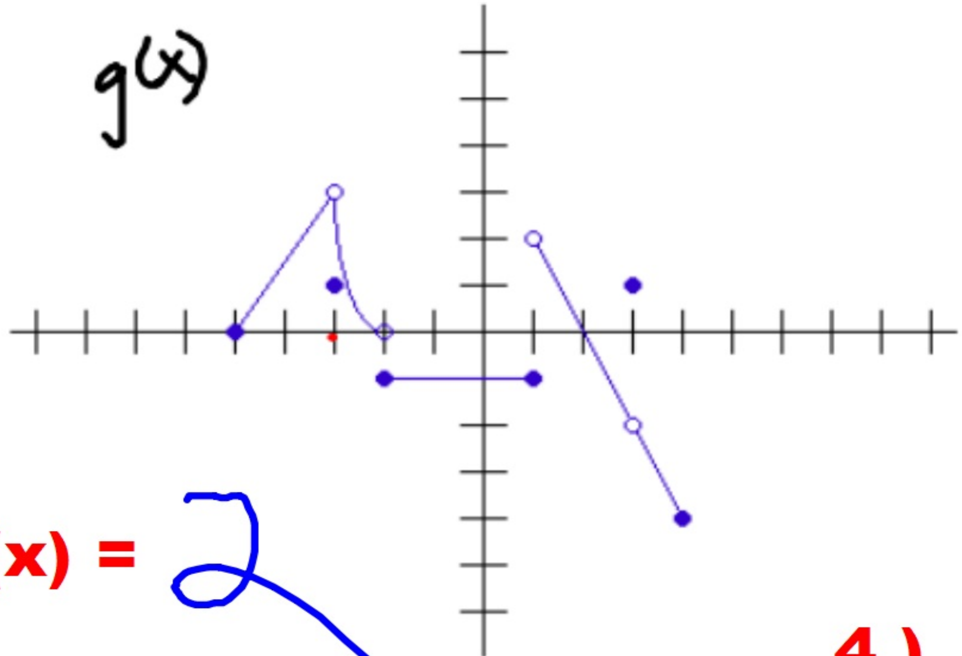
3.) $\lim_{x \to -2} f(x) =$ DNE

4.) $\lim_{x \to 2^+} f(x) =$ ∞ DNE

5.) $\lim_{x \to \infty} f(x) =$ 0

6.) $\lim_{x \to 0} f(x) =$ −1

Let g be a function defined on the interval [-5,4] whose graph is given as:



1.) $\lim_{x \to 1^{+}} g(x) =$ 2

2.) $\lim_{x \to 0} g(x) =$ -1

3.) $\lim_{x \to -3} g(x) =$ 3

4.) $\lim_{x \to 3} g(x) =$ -2

5.) $\lim_{x \to 1^{-}} g(x) =$ -1

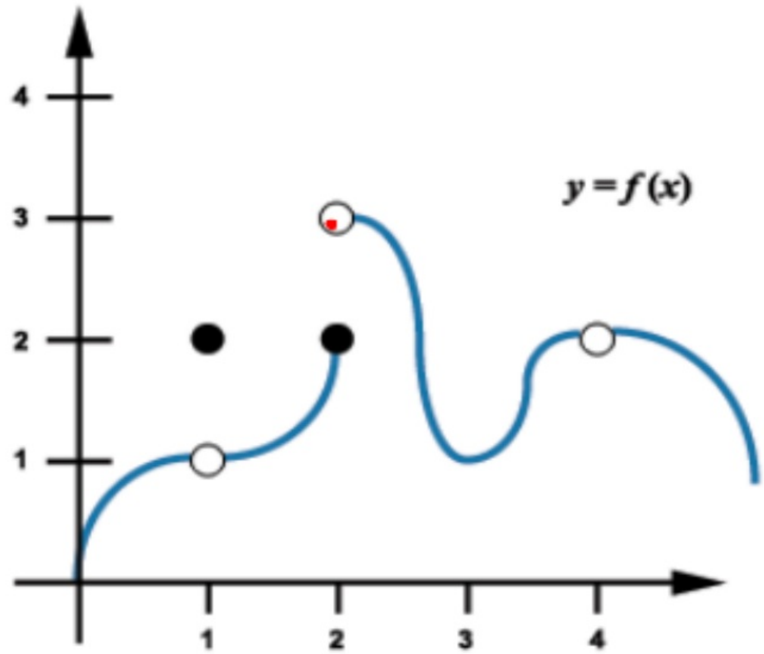
6.) $\lim_{x \to -2} g(x) =$ DNE

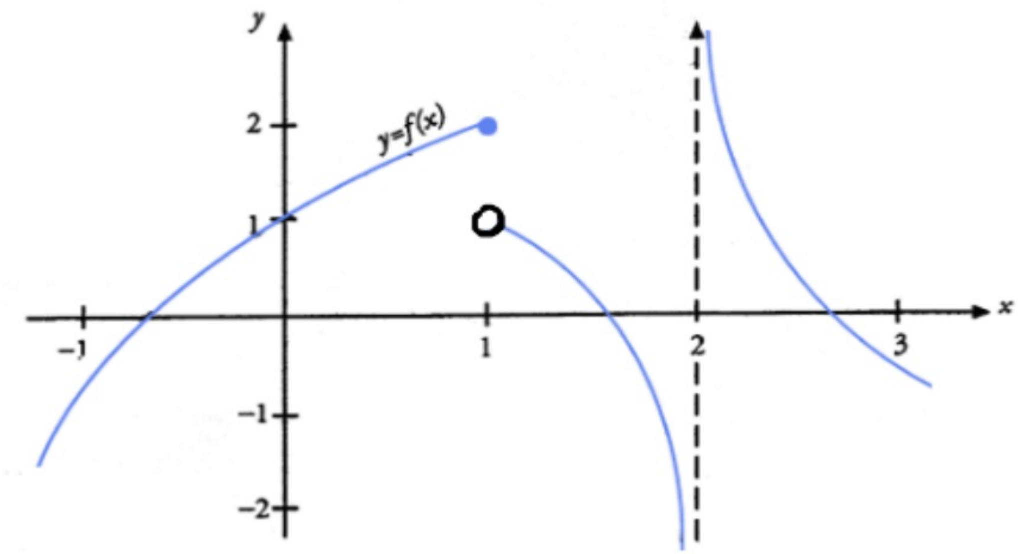
1.) $\lim_{x \to 2} f(x) =$

2.) $\lim_{x \to 4} f(x) =$

3.) $\lim_{x \to 2^+} f(x) =$

4.) $\lim_{x \to 1} f(x) =$





1.) $\lim_{x \to 0} f(x) =$

3.) $\lim_{x \to 1} f(x) =$

2.) $\lim_{x \to 2^+} f(x) =$

4.) $\lim_{x \to -\infty} f(x) =$

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} \frac{x+5}{2} = \infty$$

$$\lim_{x \to \infty} \frac{e^x}{x} = \frac{\text{Huge}}{\text{Not so Huge}} = \infty$$

$$\lim_{x \to \infty} \frac{x}{e^x} = 0$$

$$\lim_{x \to \infty} \frac{x^3}{\sqrt{x}} = \infty$$

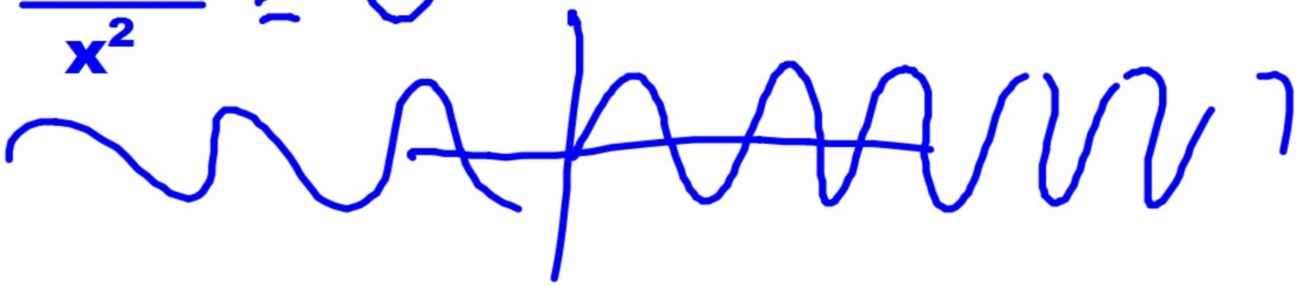
$$\lim_{x \to \infty} \frac{\ln x}{x^2} = 0$$

$$\lim_{x\to\infty} \frac{4x^2-3x+2}{5x^2-1} = \lim_{x\to\infty} \frac{4x^2}{5x^2} = \frac{4}{5}$$

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} =$$

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2}} = \frac{x}{x} = 1$$

$$\lim_{x \to \infty} \frac{\sin x}{x^2} = 0$$



$$\lim_{x\to 0} \frac{\dfrac{1}{2+x} - \dfrac{1}{\cancel{x}2}}{x}$$

$$\lim_{x\to 0} \frac{\dfrac{2-(2+x)}{2(2+x)}}{x}$$

$$\lim_{x\to 0} \frac{\dfrac{2-2-x}{2(2+x)}}{x}$$

$$\lim_{x\to 0} \frac{-\cancel{x}}{2(2+x)} \cdot \frac{1}{\cancel{x}} = \lim_{x\to 0} \frac{-1}{2(2+x)} = \frac{-1}{4}$$

$$\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} \cdot \frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}}$$

$$= \lim_{x \to 0} \frac{3 + \cancel{x} - 3}{x\left(\sqrt{3+x} + \sqrt{3}\right)}$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}\left(\sqrt{3+x} + \sqrt{3}\right)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{3+x} + \sqrt{3}} = \frac{1}{\sqrt{3} + \sqrt{3}} = \frac{1}{2\sqrt{3}}$$

$$\lim_{x \to \infty} \frac{\sin x}{x}$$

Properties of Limits as x -> +/- ∞

If L,M, and k are real numbers

1.) **Sum Rule** $\lim\limits_{x \to \pm\infty} (f(x) + g(x)) = L + M$

2.) **Difference Rule** $\lim\limits_{x \to \pm\infty} (f(x) - g(x)) = L - M$

3.) **Product Rule** $\lim\limits_{x \to \pm\infty} (f(x) * g(x)) = L * M$

4.) **Constant Multiple Rule** $\lim\limits_{x \to \pm\infty} (k * f(x) = k * L$

5.) **Quotient Rule** $\lim\limits_{x \to \pm\infty} (f(x) / g(x)) = L / M$, $M = 0$

6.) **Power Rule** $\lim\limits_{x \to \pm\infty} (f(x))^{r/s} = L^{r/s}$

Find the $\lim\limits_{x \to \infty} \dfrac{5x + \sin x}{x}$

Assignment

Page 76, numbers 5,9,27,35-38,39-44 (part b only),53,55,59-64

page 77, numbers 1-4